The general algorithm of balanced squares

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Abstract

The algorithm finded by us for generating magic squares from pozitive number allow generating magic squares any numbers including complex number. The properties of this magic (balanced) squares may open some research directions for several scientific disciplines like genetic engineering, operational research, intelligent computation, physics, etc. Taking into consideration these will describe general algoritm of balanced squares.
Key-words: magic squares, graphs, Euler cycle, algorithms, balanced squares.

1. Introduction

Let a square is divided into n^2 partial of equal cells by the help of vertical and horizontal lines of $n-1$. This square is called a square of n^{th} order.
Let the elements of integers set $\{1,2,3,...,n^2\}$ are written in the n^{th} square so that the sums of integers either on the rows or on the columns and diagonals, as well, are the same and equal to S .

$$S = \frac{n^2+1}{2}n.$$

This number is called a magic number.
If the numbers in their respective locations in the magic squares are considered as point-masses, then the mass center and geometric center a system will be coincidence. This is one of the most important properties of the magic squares. Therefore, it is advisable that to call "balanced squares" rather than "magic squares" [1].

2. The general algorithm of balanced squares

In order to understand this algorithm, where we begin to:

1. We should get 4 arithmetic sequences consist of any number;
2. We should divide the n^{th} order square to concentric frames;
3. We should get closed symmetric graphs.

These 4 arithmetic sequences will be marked α, β, γ, δ with the constants b, $c, -b, -c$, respectively. Let the initial number of the first arithmetic sequence is a_0, where a_0, b, c are any number (rational, irrational, transcendental, complex), even symbols.
Four arithmetic sequences,which have equal number of terms, form one group [2].

All the sequences of balanced square of n^{th} order consist of $\frac{n}{2}$ groups for even order or $\frac{n+1}{2}$ groups for odd order.

Let's see an example:

Let's get 4 sequences for n^{th} group of n^{th} order square, they are:

$\alpha_n - a_0, [a_0+b], [a_0+2b], ..., [a_0+(n-2)b], [a_0+(n-1)b];$

$\beta_n - [a_0+(n-1)b], [a_0+(n-1)b+c], ..., [a_0+(n-1)b+(n-2)c], [a_0+(n-1)b+(n-1)c];$

$\gamma_n - [a_0+(n-1)b+(n-1)c], [a_0+(n-2)b+(n-1)c], ..., [a_0+b+(n-1)c], [a_0+(n-1)c];$

$\delta_n - [a_0+(n-1)c], [a_0+(n-2)c], ..., [a_0+2c], [a_0+c], a_0.$

It is shown that, the sums of corresponding terms of α_n, γ_n and β_n, δ_n are equal each other. This number is constant and that is $2a_0+(n-1)(b+c)$. For examples:

$a_0+[a_0+(n-1)b+(n-1)c] = a_0+b+[a_0+(n-2)b+(n-1)c] =$

$[a_0+(n-1)b]+[a_0+(n-1)c] = ... = 2a_0+(n-1)(b+c)$.

For the other sequences groups $\{\alpha_{n-2}, \beta_{n-2}, \gamma_{n-2}, \delta_{n-2}, ..., \alpha_2, \beta_2, \gamma_2, \delta_2\}$ this value is constant, $2a_0+(n-1)(b+c)$. This constant is called first invariant the balanced square.

If we multiply the constant by $\frac{n^2}{2}$, we will obtain sum of all numbers in cells of the balanced square.

On the other hand the sum equal to nS, where S is magic number. So:

$$nS = \frac{n^2}{2}[2a_0+(n-1)(b+c)] \Rightarrow S = \frac{n}{2}[2a_0+(n-1)(b+c)].$$

If $a_0 = 1,\ b = 1,\ c = n,$ we obtain magic number for balanced square, consists of integers

$$\{1, 2, 3, ..., n^2\}, \qquad S = \frac{n^2+1}{2}n.$$

It is shown that, the sequences of n^{th} groups mentioned above, the last term of a sequence is the first term of the next. The last term of fourth sequence is the first term of the first sequence.

Note that every sequence's number of terms equal to the order groups. This fact is valid for all groups sequences.

Definition 1. A figure consisted of the n^{th} order square's outer- most cells, is called frame. The last frame left behind is a square of second order. All the frames obtained are concentric.

Theorem 1: A square of n^{th} order includes $\frac{n}{2}-1$ concentric frames and a square of second order.

If we assume that the second order square as a frame, then the frames number is equal to the number of group sequences, that is $\frac{n}{2}$.

Summary:

a)The number of cells in the frames is equal to the number of terms of groups.

b) In beginning and last terms of the sequences in each group are located on the frame corners.

2.1 THE GRAPS OF THE BALANCED DOUBLE-EVEN SQUARE OF N^{th} ORDER

Definition 2: A finite set in which some pairs of points are connected by oriented arrow (or arcs) is called an oriented graph. Points and arcs are called, respectively vertices and sides of the oriented graph. If the oriented graph the number of sides which reach a vertex is equal to the number of sides which leave it, is called a regular oriented graph. Let's denote this number by p. The regular oriented graph with $p=1$ is called regular closed oriented graph.

Theorem 2: A cycle is made, by going only one time along each side of regular graph is called closed oriented graph. This cycle is called Euler cycle [3].
In figure1 example of an Euler cycle is shown.

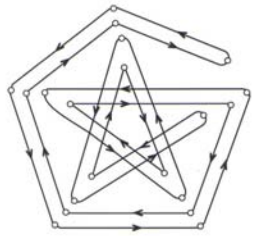

Fig.1. Euler cycle.

Here the reguler oriented closed graph, founded by Abiyev will be called Abiyev cycle.
The main idea of the method, which is presented here is to form four arithmetic sequences and write them by the Abiyev cycles into the consecutively concentric frames. Abiyev cycles depend on if the order of a frame is double-even or single-even [4]. The Abiyev cycles are enumerated in table. The sum of Abiyev cycles is equal to the sum of the concentric frames.

We write respectively for $\left\{n-2(i-1) \middle| \, i \in 1,3,5,...,\left(\frac{n}{2}-5\right)\right\}^{th}$ order frames of double-even balanced square integers of sequences will be written by the E_n Abiyev cycle, for $\left\{n-2(i-1) \middle| \, i \in 2,4,6,...,\left(\frac{n}{2}-2\right)\right\}^{th}$ order frames integers will be written by E_{n-2} Abiyev cycle. Because Abiyev cycles $E_n, E_{n-4}, E_{n-8}, ..., E_{12}$ and $E_{n-2}, E_{n-6}, E_{n-10}, ..., E_6$ are same, respectively. Abiyev cycles of 8^{th} and 4^{th} frames of square which we consider are different from E_n and Abiyev cycle of 2^{nd} order central square are different from E_{n-2}. So, Abiyev cycles E_{16}, E_{14}, E_{12}; E_{10}, E_8, E_6; and E_4, E_2 are shown in figures 2, 3, and 4, respectively.

Remark: Only the cycles of 4^{th} order balanced square are different from E_4 and E_2 for the balanced even square.
Example for $n=16$ the general balanced square is shown in figure 5.

2.2. THE GRAPS OF BALANCED SINGLE-EVEN SQUARES OF N^{th} ORDER

Now, we apply the method suggested for n^{th} order balanced double-even squares to the balanced single-even squares. Let's denote the corresponding Abiyev cycles by $E'_n, E'_{n-2}, E'_{n-4}, \dots, E'_4, E'_2$ of n^{th} order balanced square.

We write respectively the integers of sequences for $\left\{n-2(i-1) \middle| \, i \in 2, 4, 6, \dots, \left(\frac{n}{2}-1\right)\right\}^{th}$ and $\left\{n-2(i-1) \middle| \, i \in 3, 5, 7, \dots, \left(\frac{n}{2}-2\right)\right\}^{th}$ order frames which are include in single-even order balanced square with E'_{n-2} and E'_{n-4}, because $E'_{n-2}, E'_{n-6}, E'_{n-10}, \dots, E'_8$ and $E'_{n-4}, E'_{n-8}, E'_{n-12}, \dots, E'_6$ are the same between themselves.

The Abiyev cycles of the 4^{th} order frame and 2^{nd} order central square are the same in single-even and in double-even squares.

Very interesting that Abiyev cycle E'_n is different $E'_{n-4}, E'_{n-8}, \dots, E'_6$, but is same with E_{n-2} Abiyev cycle for double-even balanced square of n^{th} order. The parametres of balanced squares are summarized in table.

Table. Parametres of balanced squares.

Number of frame (i)	1	2	3	…	i	i+1	…	$\frac{n}{2}-3$	$\frac{n}{2}-2$	$\frac{n}{2}-1$	$\frac{n}{2}$
Order of frame (μ_i)	n	$n-2$	$n-4$	...	2n-2(i-1)	n-2i	…	8	6	4	2
Abiyev's cycle	E_n	E_{n-2}	E_{n-4}	…	E_μ	$E_{\mu-2}$	…	E_8	E_6	E_4	E_2
The number of cells which are in the frame	4(n-1)	4(n-3)	4(n-5)	...	4[n-(2i-1)]	4[n-(2i+1)]	…	4.7	4.5	4.3	4.1

$$\left\{1 \le i \le \frac{n}{2} \,\middle|\, i \in N\right\}$$

3. DISCUSSION:

The special properties occuring in balanced squares due to its characteristic natural phenomena. These properties, which are explained in [5], are not available in other methods [6], [7], [8], [9]. Attention, there is a perfect situation for balanced square that; the square can formed with any number.

If the Abiyev cycles are examined, this is seen that; cycles for single-even squares transform to cycles for double-even squares.

The difference between the Abiyev cycles from each other only in "+" signs.

If the cells are painted, according to sequences α, β, γ, and δ different colours, in the result the colours of cells will form 2 double butterfly wings, which are certainly different from each other.

In addition, for forming the balanced squares of odd-orders, we find the algorithm.

The 17^{th} and 15^{th} orders Abiyev cycles for odd order balanced square is shown in figure 4.

Example for n=17 the general balanced square is shown in figure 6.

THEOREM 3: The suggested algorithm allows construct the balanced square for any order [10].

The proof of this theorem will be given in another article.

4. ACKNOWLEDGEMENT

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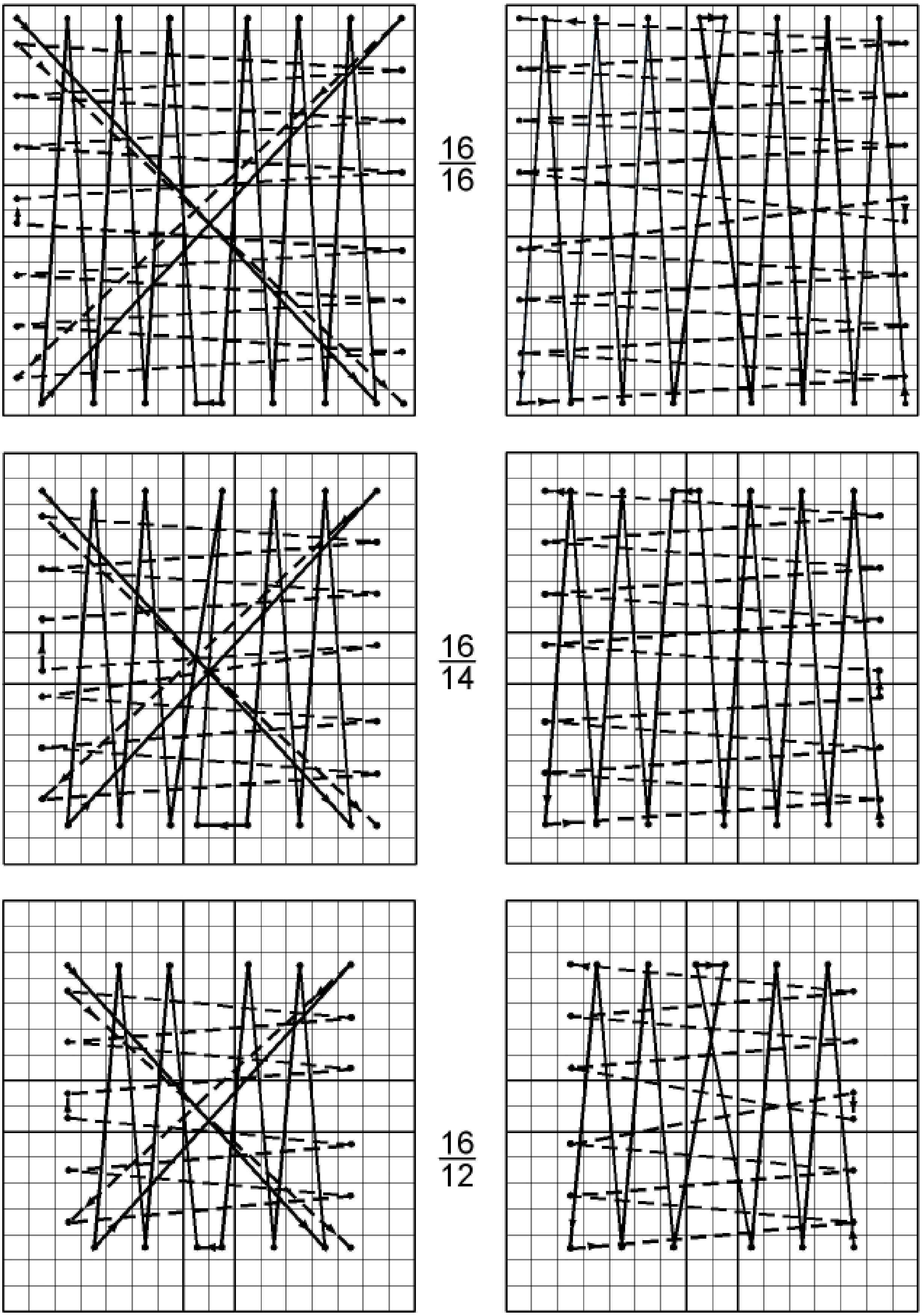

Fig. 2. 16^{th}, 14^{th}, 12^{th} orders of Abiyev cycles in 16^{th} order balanced square.

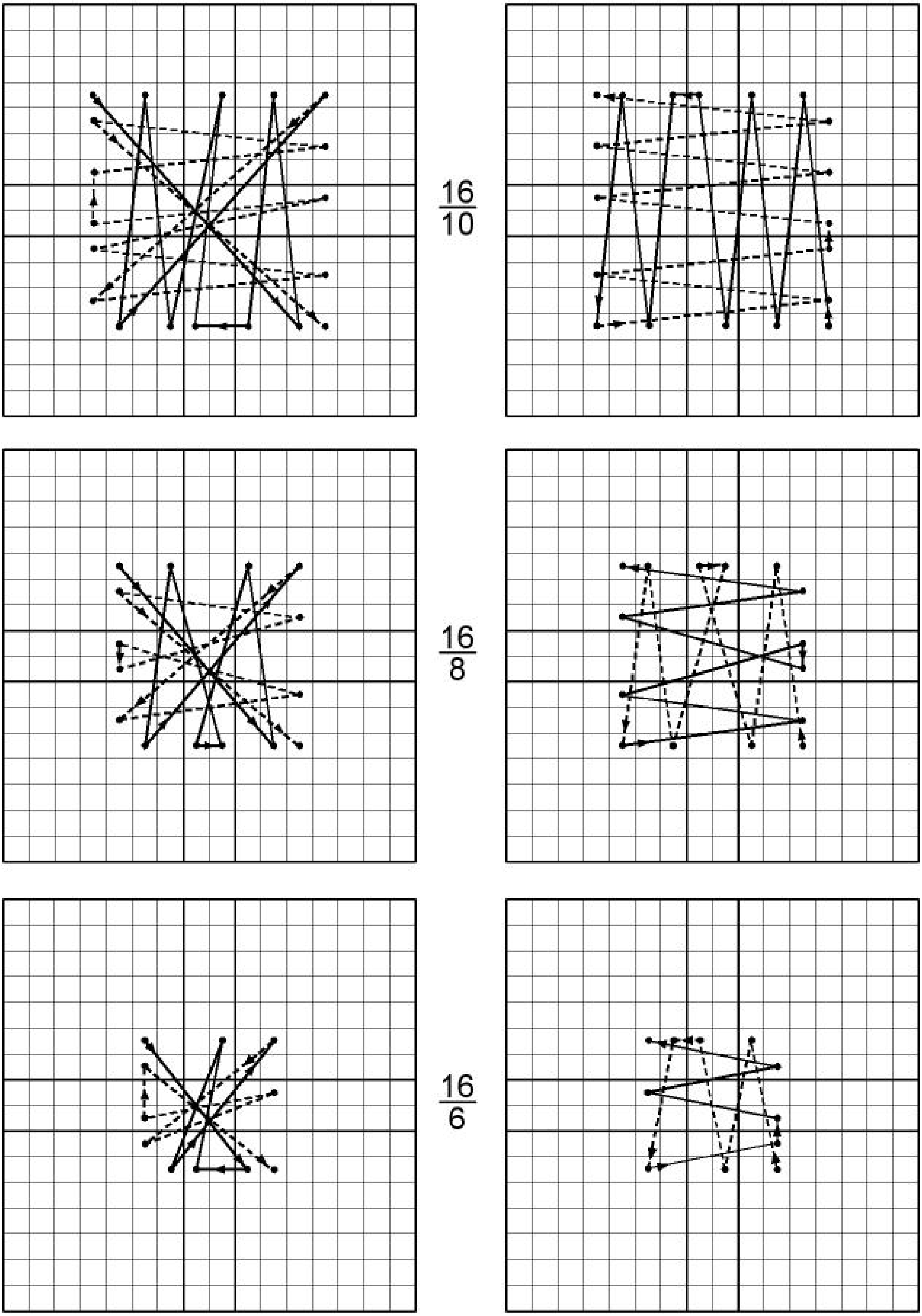

Fig. 3. 10^{th}, 8^{th}, 6^{th} orders of Abiyev cycles in 16^{th} order balanced square.

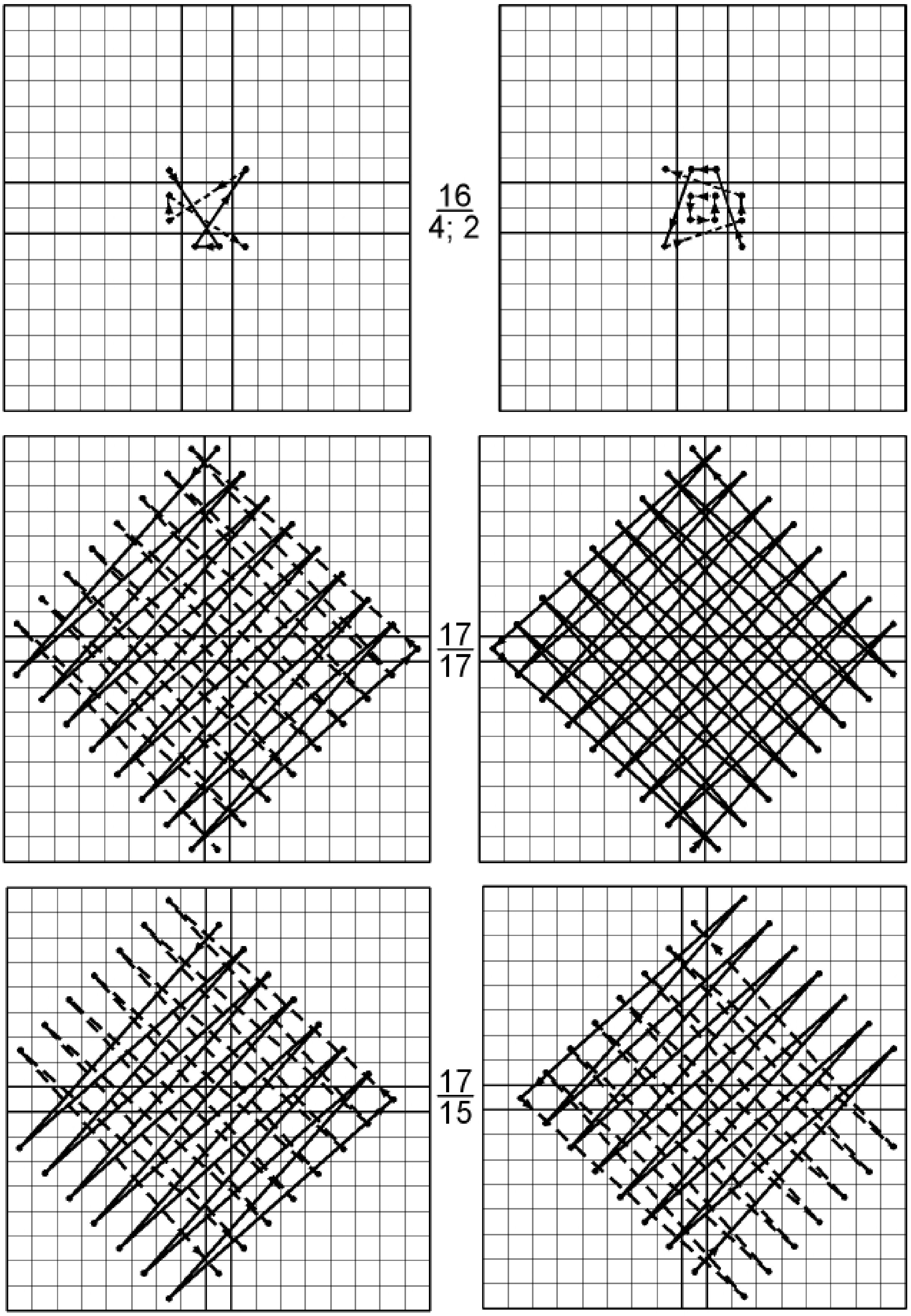

Fig. 4. 4^{th}, 2^{th} of Abiyev cycles in 16^{th} order balanced square; and 17^{th}, 15^{th} orders of Abiyev cycles in 17^{th} order balanced square.

a_0 0b 0c	1b 15c	13b 0c	3b 15c	11b 0c	5b 15c	9b 0c	8b 15c	7b 15c	6b 0c	10b 15c	4b 0c	12b 15c	2b 0c	14b 15c	15b 0c
15b 14c	1b 1c	2b 14c	12b 1c	4b 14c	10b 1c	6b 14c	7b 14c	8b 1c	9b 14c	5b 1c	11b 14c	3b 1c	13b 14c	14b 1c	0b 1c
0b 2c	14b 13c	2b 2c	3b 13c	11b 2c	5b 13c	9b 2c	8b 13c	7b 13c	6b 2c	10b 13c	4b 1c	12b 13c	13b 2c	1b 2c	15b 13c
15b 12c	1b 3c	13b 12c	3b 3c	4b 12c	10b 3c	6b 12c	7b 12c	8b 3c	9b 12c	5b 3c	11b 12c	12b 3c	2b 3c	14b 12c	0b 3c
0b 4c	14b 11c	2b 4c	12b 11c	4b 4c	5b 11c	9b 4c	8b 11c	7b 11c	6b 4c	10b 11c	11b 4c	3b 4c	13b 11c	1b 4c	15b 11c
15b 10c	1b 5c	13b 10c	3b 5c	11b 10c	5b 5c	6b 10c	7b 10c	8b 5c	9b 10c	10b 5c	4b 5c	12b 10c	2b 5c	14b 10c	0b 5c
0b 6c	14b 9c	2b 6c	12b 9c	4b 6c	10b 9c	6b 6c	7b 9c	8b 9c	9b 6c	5b 6c	11b 9c	3b 6c	13b 9c	1b 6c	15b 9c
15b 8c	1b 7c	13b 8c	3b 7c	11b 7c	5b 7c	9b 8c	8b 8c	7b 8c	6b 7c	10b 7c	4b 8c	12b 7c	2b 8c	14b 7c	0b 8c
15b 7c	14b 8c	13b 7c	12b 8c	11b 8c	10b 8c	9b 7c	8b 7c	7b 7c	6b 8c	5b 8c	4b 7c	3b 8c	2b 7c	1b 8c	0b 7c
0b 9c	14b 6c	2b 9c	12b 6c	4b 9c	10b 6c	6b 9c	8b 6c	7b 6c	9b 9c	5b 9c	11b 6c	3b 9c	13b 6c	1b 9c	15b 6c
15b 5c	1b 10c	13b 5c	3b 10c	11b 5c	5b 10c	9b 5c	7b 5c	8b 10c	6b 5c	10b 10c	4b 10c	12b 5c	2b 10c	14b 5c	0b 10c
0b 11c	14b 4c	2b 11c	12b 4c	4b 11c	10b 4c	6b 11c	7b 4c	8b 4c	9b 11c	5b 4c	11b 11c	3b 11c	13b 4c	1b 11c	15b 4c
15b 3c	1b 12c	13b 3c	3b 12c	11b 3c	5b 12c	9b 3c	7b 3c	8b 12c	6b 3c	10b 12c	4b 3c	12b 12c	2b 12c	14b 3c	0b 12c
0b 13c	14b 2c	2b 13c	12b 2c	4b 13c	10b 2c	6b 13c	8b 2c	7b 2c	9b 13c	5b 2c	11b 13c	3b 2c	13b 13c	1b 13c	15b 2c
15b 1c	1b 14c	13b 1c	3b 14c	11b 1c	5b 14c	9b 1c	7b 1c	8b 14c	6b 1c	10b 14c	4b 1c	12b 14c	2b 1c	14b 14c	0b 14c
0b 15c	14b 0c	2b 15c	12b 0c	4b 15c	10b 0c	6b 15c	8b 0c	7b 0c	9b 15c	5b 0c	11b 15c	3b 0c	13b 15c	1b 0c	15b 15c

Fig. 5. General balanced square of 16^{th} order.
The first term (a_0) of the sequences are present in all cells.

9b 8c	10b 7c	11b 6c	12b 5c	13b 4c	14b 3c	15b 2c	16b 1c	a_0 0b 0c	1b 16c	2b 15c	3b 14c	4b 13c	5b 12c	6b 11c	7b 10c	8b 9c
10b 9c	11b 8c	12b 7c	13b 6c	14b 5c	15b 4c	16b 3c	0b 2c	1b 1c	2b 0c	3b 16c	4b 15c	5b 14c	6b 13c	7b 12c	8b 11c	9b 10c
11b 10c	12b 9c	13b 8c	14b 7c	15b 6c	16b 5c	0b 4c	1b 3c	2b 2c	3b 1c	4b 0c	5b 16c	6b 15c	7b 14c	8b 13c	9b 12c	10b 11c
12b 11c	13b 10c	14b 9c	15b 8c	16b 7c	0b 6c	1b 5c	2b 4c	3b 3c	4b 2c	5b 1c	6b 0c	7b 16c	8b 15c	9b 14c	10b 13c	11b 12c
13b 12c	14b 11c	15b 10c	16b 9c	0b 8c	1b 7c	2b 6c	3b 5c	4b 4c	5b 3c	6b 2c	7b 1c	8b 0c	9b 16c	10b 15c	11b 14c	12b 13c
14b 13c	15b 12c	16b 11c	0b 10c	1b 9c	2b 8c	3b 7c	4b 6c	5b 5c	6b 4c	7b 3c	8b 2c	9b 1c	10b 0c	11b 16c	12b 15c	13b 14c
15b 14c	16b 13c	0b 12c	1b 11c	2b 10c	3b 9c	4b 8c	5b 7c	6b 6c	7b 5c	8b 4c	9b 3c	10b 2c	11b 1c	12b 0c	13b 16c	14b 15c
16b 15c	0b 14c	1b 13c	2b 12c	3b 11c	4b 10c	5b 9c	6b 8c	7b 7c	8b 6c	9b 5c	10b 4c	11b 3c	12b 2c	13b 1c	14b 0c	15b 16c
0b 16c	1b 15c	2b 14c	3b 13c	4b 12c	5b 11c	6b 10c	7b 9c	8b • 8c	9b 7c	10b 6c	11b 5c	12b 4c	13b 3c	14b 2c	15b 1c	16b 0c
1b 0c	2b 16c	3b 15c	4b 14c	5b 13c	6b 12c	7b 11c	8b 10c	9b 9c	10b 8c	11b 7c	12b 6c	13b 5c	14b 4c	15b 3c	16b 2c	0b 1c
2b 1c	3b 0c	4b 16c	5b 15c	6b 14c	7b 13c	8b 12c	9b 11c	10b 10c	11b 9c	12b 8c	13b 7c	14b 6c	15b 5c	16b 4c	0b 3c	1b 2c
3b 2c	4b 1c	5b 0c	6b 16c	7b 15c	8b 14c	9b 13c	10b 12c	11b 11c	12b 10c	13b 9c	14b 8c	15b 7c	16b 6c	0b 5c	1b 4c	2b 3c
4b 3c	5b 2c	6b 1c	7b 0c	8b 16c	9b 15c	10b 14c	11b 13c	12b 12c	13b 11c	14b 10c	15b 9c	16b 8c	0b 7c	1b 6c	2b 5c	3b 4c
5b 4c	6b 3c	7b 2c	8b 1c	9b 0c	10b 16c	11b 15c	12b 14c	13b 13c	14b 12c	15b 11c	16b 10c	0b 9c	1b 8c	2b 7c	3b 6c	4b 5c
6b 5c	7b 4c	8b 3c	9b 2c	10b 1c	11b 0c	12b 16c	13b 15c	14b 14c	15b 13c	16b 12c	0b 11c	1b 10c	2b 9c	3b 8c	4b 7c	5b 6c
7b 6c	8b 5c	9b 4c	10b 3c	11b 2c	12b 1c	13b 0c	14b 16c	15b 15c	16b 14c	0b 13c	1b 12c	2b 11c	3b 10c	4b 9c	5b 8c	6b 7c
8b 7c	9b 6c	10b 5c	11b 4c	12b 3c	13b 2c	14b 1c	15b 0c	16b 16c	0b 15c	1b 14c	2b 13c	3b 12c	4b 11c	5b 10c	6b 9c	7b 8c

Fig. 6. General balanced square of 17th order.
The first term (a_0) of the sequences are present in all cells.

a_0 0b 0c	1b 17c	15b 0c	3b 17c	13b 0c	5b 17c	11b 0c	7b 17c	8b 17c	9b 0c	10b 17c	6b 0c	12b 17c	4b 0c	14b 17c	2b 0c	16b 17c	17b 0c
17b 16c	1b 1c	2b 16c	14b 1c	4b 16c	12b 1c	6b 16c	10b 1c	8b 16c	9b 16c	7b 1c	11b 16c	5b 1c	13b 16c	3b 1c	15b 16c	16b 1c	0b 1c
0b 2c	16b 15c	2b 2c	3b 15c	13b 2c	5b 15c	11b 2c	7b 15c	9b 2c	8b 15c	10b 15c	6b 2c	12b 15c	4b 2c	14b 15c	15b 2c	1b 2c	17b 15c
17b 14c	1b 3c	15b 14c	3b 3c	4b 14c	12b 3c	6b 14c	10b 3c	8b 14c	9b 14c	7b 3c	11b 14c	5b 3c	13b 14c	14b 3c	2b 3c	16b 14c	0b 3c
0b 4c	16b 13c	2b 4c	14b 13c	4b 4c	5b 13c	11b 4c	7b 13c	9b 4c	8b 13c	10b 13c	6b 4c	12b 13c	13b 4c	3b 4c	15b 13c	1b 4c	17b 13c
17b 12c	1b 5c	15b 12c	3b 5c	13b 12c	5b 5c	6b 12c	10b 5c	8b 12c	9b 12c	7b 5c	11b 12c	12b 5c	4b 5c	14b 12c	2b 5c	16b 12c	0b 5c
0b 6c	16b 11c	2b 6c	14b 11c	4b 6c	12b 11c	6b 6c	7b 11c	9b 6c	8b 11c	10b 11c	11b 6c	5b 6c	13b 11c	3b 6c	15b 11c	1b 6c	17b 11c
17b 10c	1b 7c	15b 10c	3b 7c	13b 10c	5b 7c	11b 10c	7b 7c	8b 10c	9b 10c	10b 7c	6b 7c	12b 10c	4b 7c	14b 10c	2b 7c	16b 10c	0b 7c
0b 8c	16b 8c	15b 9c	14b 8c	13b 9c	12b 8c	11b 9c	10b 9c	9b 9c	8b 9c	7b 8c	6b 9c	5b 8c	4b 9c	3b 8c	2b 9c	1b 8c	17b 8c
17b 9c	16b 9c	2b 8c	14b 9c	4b 8c	12b 9c	6b 8c	10b 8c	9b 8c	8b 8c	7b 9c	11b 8c	5b 9c	13b 8c	3b 9c	15b 8c	1b 9c	0b 9c
17b 7c	1b 10c	15b 7c	3b 10c	13b 7c	5b 10c	11b 7c	7b 10c	9b 7c	8b 7c	10b 10c	6b 10c	12b 7c	4b 10c	14b 7c	2b 10c	16b 7c	0b 10c
0b 11c	16b 6c	2b 11c	14b 6c	4b 11c	12b 6c	6b 11c	10b 6c	9b 11c	8b 6c	7b 6c	11b 11c	5b 11c	13b 6c	3b 11c	15b 6c	1b 11c	17b 6c
17b 5c	1b 12c	15b 5c	3b 12c	13b 5c	5b 12c	11b 5c	7b 12c	8b 5c	9b 5c	10b 12c	6b 5c	12b 12c	4b 12c	14b 5c	2b 12c	16b 5c	0b 12c
0b 13c	16b 4c	2b 13c	14b 4c	4b 13c	12b 4c	6b 13c	10b 4c	9b 13c	8b 4c	7b 4c	11b 13c	5b 4c	13b 13c	3b 13c	15b 4c	1b 13c	17b 4c
17b 3c	1b 14c	15b 3c	3b 14c	13b 3c	5b 14c	11b 3c	7b 14c	8b 3c	9b 3c	10b 14c	6b 3c	12b 14c	4b 3c	14b 14c	2b 14c	16b 3c	0b 14c
0b 15c	16b 2c	2b 15c	14b 2c	4b 15c	12b 2c	6b 15c	10b 2c	9b 15c	8b 2c	7b 2c	11b 15c	5b 2c	13b 15c	3b 2c	15b 15c	1b 15c	17b 2c
17b 1c	1b 16c	15b 1c	3b 16c	13b 1c	5b 16c	11b 1c	7b 16c	8b 1c	9b 1c	10b 16c	6b 1c	12b 16c	4b 1c	14b 16c	2b 1c	16b 16c	0b 16c
0b 17c	16b 0c	2b 17c	14b 0c	4b 17c	12b 0c	6b 17c	10b 0c	8b 0c	9b 17c	7b 0c	11b 17c	5b 0c	13b 17c	3b 0c	15b 17c	1b 0c	17b 17c

Fig. 7. General balanced square of 18th order.
The first term (a_0) of the sequences are present in all cells.